MAN-003-001617-1250-4

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Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2018

Mathematics : Paper - 602 (A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003

Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following : **20**

(1) Define Least Upper Bound

(2) Define Countable set

(3) Define Totally Bounded set

(4) Is Q (Set of Rational) compact set?

(5) Is the arbitrary in trisection of compact set compact?

(6) L $\left(t^{-1/2}\right) =$

(7) Define is $L^{-1}\left(\frac{3}{(s-1)^2-9}\right) =$

(8) Define convolution function

(9) $L\left(f\left(t\right)\right) = f^{-}\left(s\right)$ then $L\left[\int_0^1 f\left(u\right)du\right] =$

(10) $L^{-1}\left(\frac{1}{4s+5}\right) =$

(11) Define Epimorphism

(12) How many proper Ideal can a field have?

(13) Define Leading Coefficient

(14) Define Divisor of Polynomial

(15) Find characteristic of the ring $(Z_8, +_8, *_8)$

(16) Find the greatest lower bound of $\left\{\frac{1}{n} / n \in N\right\}$

(17) Find $L^{-}\left(\frac{s}{\left(s^2+a^2\right)^2}\right) =$

(18) If polynomial $f = (3, 0, 0, 0, ...)$ then find $|f|$

(19) Find zero divisor of $(Z_6, +_6, *6)$

(20) What do you mean by Cubic polynomials ?

2 (A) Attempt any **Three** : **6**

(1) $E_n = [-n, n]\ n \in N$ then the collection $\{E_n \mid n \in N\}$ is a cover of R or not ?

(2) Show that $A = (1, 2)$ and $B = [2, 3)$ are not Separated sets of Metric space R.

(3) State and prove Heine – Borel Theorem.

(4) Find Laplace Transformation of $f(t) = \left\{ \begin{matrix} t, 0<t<4 \\ 5, t>4 \end{matrix} \right\}$

(5) Prove If L $L\{f(t)\} = \overline{f(s)}$ then $L\left[\int_0^t f(u)\,du\right] = \frac{1}{8}\overline{f(s)}$

(6) Find Inverse Laplace Transformation of $\frac{s}{\left(s^2-1\right)^2}$

(B) Attempt any **Three** : **9**

(1) Let E be a non-empty closed subset of metric space R. If E is upper bounded set then Iub E lies in E.

(2) By using definition prove that $(0, 3)$ is not compact.

(3) Let (X, d) be a metric space. Then X is totally bounded set.

(4) Find Laplace transformation of $^{-3t}\sin^2 t$

(5) Let $L\{f(t)\} = \overline{f(s)}$ and $\left(\frac{f(t)}{t}\right)$ has Laplace transform then $L\left\{\frac{f(t)}{t}\right\} = \int_0^{\infty} f(\overline{s})\, ds$

(6) Find $L^{-1}\left\{\log\left(1+\frac{4}{s^2}\right)\right\}$

(C) Attempt any **Two** : **10**

(1) Prove that every closed subset of compact set is compact in metric space.

(2) Every Compact set of a metric space is closed and Bounded

(3) A metric space (X, d) is a sequential compact if and only if it satisfies Bolzano–weirstrass theorem

(4) Using convolution theorem. Find inverse Laplace Transformation of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$

(5) Find Inverse Laplace transformation of $\frac{S}{s^4+s^2+1}$

3 (A) Attempt any **Three** : **6**

(1) $\Phi : \left(G, ^{*}\right) \rightarrow (G', \Delta)$ be Homomorphism, If N is a normal subgroup of G then $\Phi(N)$ is a Normal subgroup of $\Phi(G)$.

(2) Show that a cyclic group of order eight is homorphism to a cyclic group of order four.

(3) Obtain radical of the ring $(Z_{12}, +_{12}, \bullet_{12})$ and $(Z_{19}, +_{19}, \bullet_{19})$

(4) Prove that field has no proper Ideal.

(5) $f(x) = (2, 0, -3.0.4, 0, 0, 0....)$ and $g(x) = (1, -2, 0, 0, 0....) \in R[x]$ them find $f(x).\, g(x)$

(6) Factorize $f(x) = x^4 + 4 \in Z_5[x]$ by using factor theorem.

(B) Attempt any **Three** : **9**

(1) What will be the intersection of a right and left Ideals of a ring ? Justify

(2) Let $\Phi:(G,*)\rightarrow(G',\Delta)$ be Homomorphism, Then K_{ϕ} Is a normal subgroup of G.

(3) Is $U=\left\{f\in C[0,1]/\int_{0}^{1}f(t)\,dt=1\right\}$ a subring of $(C[0,1],+,*)$?

(4) Prove that field has no proper Ideal.

(5) Find g.c.d. of $f(x)=6x^{3}+5x^{2}-2x+25$ and $g(x)=2x^{2}-3x+5\in R[x]$

(6) State and prove Remainder theorem

(C) Attempt any **Two** : **10**

(1) A Homomorphism $\Phi:(G,*)\rightarrow(G',\Delta)$ is one-one if and if only $K_{\phi}=\{e\}$

(2) A commutative ring with unity is a field if it has no Proper Ideal.

(3) A non-empty subset I of a ring R is an Ideal of R. iff the following two conditions hold.

(a) $a-b\in I \quad \forall a,b\in I$

(b) $ra, ar\in I, \quad \forall a\in R$

(4) Express $f(x)$ as $q(x)g(x)+r(x)$ form by using division algorithm for

$$\left.\begin{array}{l}f(x)=x^{4}-3x^{3}+2x^{2}+4x-1\\ g(x)=x^{2}-2x+3\end{array}\right\}\in Z_{5}[x]$$

(5) Any Ideal in integral domain $f(x)$ is a Principal Ideal.